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Cox's design in repeated row-column design with split plots and a control

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Summary

The paper deals with two cases of repeated row-column designs with split plots in which either one of the whole plot or one of the subplot treatments is a control treatment.

In the first case considered, with control on whole plots, the row-column design consists of two rows and two columns (4 whole plots) and each whole plot is complete with respect to subplot treatments. One of the test treatments and the (whole plot) control treatment occur once in a row and once in a column.

In the second case, the whole plot treatments are arranged in a Latin square and each whole plot has only two subplots. We assume that one of the subplot test treatments and subplot control occur on each whole plot.

To improve statistical properties of the design we recommend to repeat the whole design t times.

In the paper we give a characterization of these designs with respect to the general balance property. Moreover, we characterize the design with respect to the efficiency balance.

1. Introduction

Let us consider an experiment utilizing the material with structure described below. An experimental material is divided into k_1 rows and k_2 columns, similarly as in a row-column design. The plot being an intersection of a row and a column is called the whole plot. This means that we consider ordinary row column designs only. Let each whole plot then be divided into k_3 subplots. The experimental material of the above structure will be called a superblock. A two-factorial

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Key words: row-column designs, incomplete design, control treatments, efficiency factors.

experiment with k_0 superblocks, in which levels of one factor, A, are arranged on the whole plots and levels of the second factor, B, are arranged on the subplots, has been described in detail previously (Kachlicka and Mejza, 1990, 1995).

Two cases, interesting for an experimenter, are considered in this paper. In the first case one of the levels of factor A, say A_0, is a control treatment (or standard). The other levels of factor A are called test treatments (or whole plot test treatments). One of the levels of factor B is a control treatment (or standard) in the second case. Similarly, the other levels of factor B are called test treatments (or, more exactly, subplot test treatments). The purpose of the experiment is to compare test treatments with the control as precisely as possible for a given structure of the experimental design. In this paper we restrict our considerations to the (potential) experimental material, where only two plots are available to the treatments including the control. There are some ways to arrange test and control treatments on the blocks of size two. The recommended design, known as the Cox design, (cf. Cox, 1958) is such that (within each block) the control occurs on one plot and one of test treatments on the other. In the paper we give a characterization of the Cox type design, for whole plot treatments or subplot treatments, with respect to its efficiency. More exactly, the designs considered are complete with respect to the treatments not including the control, while they are incomplete with respect to treatments including the control. Two possible cases of such a design are examined.

The Cox's type of design is usually used in the situation where only two natural experimental units are available. For example, in the experiments dealing with leaves we can situate units on the left and right part of the leaf. Many biological and medical experiments deal with twins. Then the proposed design is recommended. Moreover, the proposed design is useful also in experiments utilizing double organs, e.g. kidneys, eyes, or two symmetrical parts of the body, e.g. hands, legs.

The designs considered may be very useful in factorial experiments in which one of the factors (e.g. whole plot treatments) is related to irrigation. In this case, it may be convenient to design an experiment in such a way that on whole plots no irrigation as well as one of the levels of irrigation is applied. The different levels of fertilization would constitute, for example, the subplot treatments.

2. Experiment with A_0 as a control treatment.

Let us consider a two-factorial experiment in which one of $a+1$ levels of factor A is the control level A_0 (whole plot control).

Moreover, let us assume that k_0 superblocks are composed of two rows and two columns, i.e. $k_1 = k_2 = 2$. The control treatment A_0 occurs on one whole plot within each row and each column and one of the remaining levels of A occurs on the other whole plot. Each whole plot is divided into b subplots on which all levels of factor B (subplot treatments) are arranged ($k_3 = b$). The minimal design has a superblock, in which the whole plot test treatments are replicated 2 times while the control – $2a$ times. This design is repeated t times ($k_0 = at$) to improve statistical properties of estimators.

For example, a superblock in which the control treatment A_0 and one test treatment occur, may be schematically expressed as follows:

A_0				A_i			
B_1	B_2	...	B_b	B_1	B_2	...	B_b
A_i				A_0			
B_1	B_2	...	B_b	B_1	B_2	...	B_b

The number of treatments (treatment combinations) is equal to $v = (a + 1)b$ while the treatment replication vector has the form

$$\mathbf{r} = 2t\begin{bmatrix} a \\ \mathbf{1}_a \end{bmatrix} \otimes \mathbf{1}_b ,$$

where $\otimes$ denotes the Kronecker product of matrices.

Since the described design has the orthogonal block structure (according to Nelder, 1965 and Houtman and Speed, 1983) the total information on treatment comparisons may be divided into five independent strata. The strata are as follows: zero stratum (called also total area stratum); first, inter-superblock stratum; second, inter-row stratum; third, inter-column stratum; fourth, inter-whole plots stratum; and fifth, inter-subplot stratum. In fact there are six strata but the information concerning treatment contrasts is contained in non zero strata. The strata are connected with randomization performed in the experiment. In the paper we deal with the linear model resulting from randomization of superblocks, rows, columns, whole plots and subplots. (For details concerning this model the reader is referred to Kachlicka and Mejza, 1990, 1995).

The analysis of variance appropriate for multistratum experiments with the orthogonal block structure is based on the so called information matrices for the strata. To find them, let us define the incidence matrices $\mathbf{N}_i$, $i = 1,2,3,4$, for superblocks vs treatments, rows vs treatments, columns vs treatments and whole plots vs treatments. These incidence matrices have the forms:

$$\mathbf{N}_1 = 2\, \mathbf{1}_t' \otimes \begin{bmatrix} \mathbf{1}_a' \\ \mathbf{I}_a \end{bmatrix} \otimes \mathbf{1}_b ,$$

$$\mathbf{N}_2 = \mathbf{1}'_t \otimes \begin{bmatrix} \mathbf{1}'_a \\ \mathbf{I}_a \end{bmatrix} \otimes \mathbf{1}'_2 \otimes \mathbf{1}_b \ ,$$

$$\mathbf{N}_3 = \mathbf{1}'_t \otimes \begin{bmatrix} \mathbf{1}'_a \\ \mathbf{I}_a \end{bmatrix} \otimes \mathbf{1}'_2 \otimes \mathbf{1}_b \ ,$$

$$\mathbf{N}_4 = \mathbf{1}'_t \otimes \begin{bmatrix} \mathbf{1}'_a \otimes \ [1\ 0\ 0\ 1] \\ \mathbf{I}_a \otimes \ [0\ 1\ 1\ 0] \end{bmatrix} \otimes \mathbf{1}_b \ .$$

The strata information matrices are expressible in terms of association matrices, which for the considered design have the following forms:

$$\mathbf{N}_1\mathbf{N}'_1 = 4t \begin{bmatrix} a & \mathbf{1}'_a \\ \mathbf{1}_a & \mathbf{I}_a \end{bmatrix} \otimes \mathbf{J}_b \ ,$$

$$\mathbf{N}_2\mathbf{N}'_2 = 2t \begin{bmatrix} a & \mathbf{1}'_a \\ \mathbf{1}_a & \mathbf{I}_a \end{bmatrix} \otimes \mathbf{J}_b \ ,$$

$$\mathbf{N}_3\mathbf{N}'_3 = \mathbf{N}_2\mathbf{N}'_2 \ ,$$

$$\mathbf{N}_4\mathbf{N}'_4 = 2t \begin{bmatrix} a & \mathbf{0}'_a \\ \mathbf{0}_a & \mathbf{I}_a \end{bmatrix} \otimes \mathbf{J}_b = \mathbf{r}^{\delta}_A \otimes \mathbf{J}_b \ , \ \text{where} \ \mathbf{J}_b = \mathbf{1}_b\mathbf{1}'_b .$$

The statistical properties of a design are connected with the structure of the so called **C** matrices, associated with the blocks of the design, i.e. where superblocks, rows, columns and whole plots play the role of blocks.

Now, the C-type matrices, $\mathbf{C}_i$, $i = 1,2,3,4,5$, may be expressed as follows:

$$\mathbf{C}_1 = (k_1k_2k_3)^{-1}\mathbf{N}_1\mathbf{N}'_1 - (k_0k_1k_2k_3)^{-1}\mathbf{r}\mathbf{r}' = \frac{t}{b}\begin{bmatrix} 0 & \mathbf{0}'_a \\ \mathbf{0}_a & \mathbf{I}_a - a^{-1}\mathbf{J}_a \end{bmatrix} ,$$

$$\mathbf{C}_2 = (k_2k_3)^{-1}\mathbf{N}_2\mathbf{N}'_2 - (k_1k_2k_3)^{-1}\mathbf{N}_1\mathbf{N}'_1 = \mathbf{0} \ ,$$

$$\mathbf{C}_3 = (k_1k_3)^{-1}\mathbf{N}_3\mathbf{N}'_3 - (k_1k_2k_3)^{-1}\mathbf{N}_1\mathbf{N}'_1 = \mathbf{0} \ ,$$

$$\mathbf{C}_4 = k_3^{-1}\mathbf{N}_4\mathbf{N}'_4 - (k_2k_3)^{-1}\mathbf{N}_3\mathbf{N}'_3 - (k_1k_3)^{-1}\mathbf{N}_2\mathbf{N}'_2 + (k_1k_2k_3)^{-1}\mathbf{N}_1\mathbf{N}'_1 =$$

$$= \frac{t}{b}\begin{bmatrix} a & -\mathbf{1}'_a \\ -\mathbf{1}_a & \mathbf{I}_a \end{bmatrix} \otimes \mathbf{J}_b \ ,$$

$$\mathbf{C}_5 = \mathbf{r}^{\delta} - k_3^{-1}\mathbf{N}_4\mathbf{N}'_4 = \mathbf{r}^{\delta}_A \otimes (\mathbf{I}_b - b^{-1}\mathbf{J}_b) \ .$$

An additional property, called general balance, is very helpful in planning multistratum experiments, and simplifies drawing conclusions (for details the reader is referred to Houtman and Speed, 1983). It is easy to check, that the matrices $\mathbf{C}_i$ mutually commute with respect to (w.r.t.) $\mathbf{r}^{-\delta}$, i.e.

$$\mathbf{C}_i\mathbf{r}^{-\delta}\mathbf{C}_{i'} = \mathbf{C}_{i'}\mathbf{r}^{-\delta}\mathbf{C}_i \ , \qquad i,i' = 1,2,3,4,5, \quad i \neq i' \ .$$

This means that the considered design is generally balanced (cf. Mejza, 1992). From this property one can conclude that the matrices $\mathbf{C}_i$, $i = 1,2,3,4,5$, have a common set of $\mathbf{r}^{\delta}$-orthonormal eigenvectors $\mathbf{p}_j$ ($j = 1,2,\ldots,v$), i.e. $\mathbf{p}'_j\mathbf{r}^{\delta}\mathbf{p}_{j'} = \delta_{jj'}$, where $\delta_{jj'}$ is Kronecker's delta.

One of possible sets of such eigenvectors may be found as follows:

$$\mathbf{p}_j = \mathbf{p}_{A_h} \otimes \mathbf{p}_{B_i} \ , \qquad h = 0,1,\ldots,a, \quad i = 1,2,\ldots,b,$$

$$\mathbf{p}'_{A_0} = 1/\sqrt{a+1}\ [1 \ \ -1 \ \ -1 \ \ldots \ -1 \ \ -1] \ ,$$

$$\mathbf{p}'_{A_1} = 1/\sqrt{2}\ [0 \ \ 1 \ \ -1 \ \ldots \ 0 \ \ 0] \ ,$$

$$\ldots$$

$$\mathbf{p}'_{A_{a-1}} = 1/\sqrt{a(a-1)}\ [0 \ \ 1 \ \ 1 \ \ldots \ 1 \ \ -(a-1)] \ ,$$

$$\mathbf{p}'_{A_a} = 1/\sqrt{a+1}\ [1 \ \ 1 \ \ 1 \ \ldots \ 1 \ \ 1] \ ,$$

$$\mathbf{p}'_{B_1} = 1/\sqrt{2}\ [1 \ \ -1 \ \ 0 \ \ldots \ 0] \ ,$$

$$\mathbf{p}'_{B_2} = 1/2\ [1 \ \ 1 \ \ -2 \ \ldots \ 0] \ ,$$

$$\ldots$$

$$\mathbf{p}'_{B_{b-1}} = 1/\sqrt{b(b-1)}\ [1 \ \ 1 \ \ 1 \ \ldots \ -(b-1)] \ ,$$

$$\mathbf{p}'_{B_b} = 1/\sqrt{b}\ [1 \ \ 1 \ \ldots \ 1] \ .$$

Let ε_{ji} denote the eigenvalue of matrix $\mathbf{C}_i$ w.r.t. $\mathbf{r}^{\delta}$, $j = 1,2,\ldots,v$, $i = 1,2,3,4,5$. Note that $\mathbf{C}_i\mathbf{1} = \mathbf{0}$, and hence $\mathbf{p}'_j\mathbf{r}^{\delta}\mathbf{1} = 0$, i.e. the vectors $\mathbf{p}'_j\mathbf{r}^{\delta}$ define a contrast $\mathbf{p}'_j\mathbf{r}^{\delta}\boldsymbol{\gamma}$, which is called the basic contrast, $j = 1,2,\ldots,v-1$ ($\boldsymbol{\gamma}$ denotes the vector of treatment combination parameters). From the equality

$$\sum_i \varepsilon_{ji} = 1 \ , \quad \text{for all } j<v \ ,$$

it results that the eigenvalues ε_{ji} may be identified as the i-th stratum efficiency factors of the design with respect to the basic contrast $\mathbf{p}'_j\mathbf{r}^{\delta}\boldsymbol{\gamma}$.

The structure of matrices $\mathbf{C}_i$, $i = 1,2,3,4,5$, shows that their eigenvalues w.r.t. $\mathbf{r}^{\delta}$ are as follows:

for $\mathbf{C}_1$: $a-1$ eigenvalues are equal to 0, i.e. $\varepsilon = 0.5$ and the remaining $(a+1)(b-1)+2$ eigenvalues are equal to 0, i.e. $\varepsilon = 0$,

for $\mathbf{C}_2$ and $\mathbf{C}_3$: all eigenvalues are equal to 0, i.e. $\varepsilon = 0$,

for $\mathbf{C}_4$: one eigenvalue is equal to 1, i.e $\varepsilon = 1$; $a-1$ eigenvalues are equal to 0, i.e. $\varepsilon = 0.5$ and the remaining $(a+1)(b-1)+1$ eigenvalues are equal to 0, i.e. $\varepsilon = 0$, for C_5: $(a+1)(b-1)$ eigenvalues are equal to 1, i.e. $\varepsilon = 1$; the remaining $a+1$ eigenvalues are equal to 0, i.e. $\varepsilon = 0$.

Thus, the structure of C-matrices indicates that:

(i) no contrasts are estimated in the second and third strata;

(ii) all the contrasts concerning the levels of factor B and interaction $A \times B$ are estimated with the full efficiency ($\varepsilon = 1$) in the fifth stratum;

(iii) contrast enabling comparison of the control treatment A_0 with the mean effect of all test treatments can be estimated with the full efficiency in the fourth stratum;

(iv) elementary contrasts among test treatments A_i and the control are estimated in the fifth stratum with efficiency equal to $(a+1)/2a$,

(v) the contrasts among test treatments A only are estimated in the first and fourth strata with the same efficiency factors equal to 0.5.

3. Experiment with B_0 as a subplot control treatment

In this section we consider a two-factorial experiment in which one of $b+1$ levels of factor B is the (subplot) control treatment, B_0. According to the methodology of the paper, the design is complete with respect to the whole plot treatments, i.e. a superblock may consist of a rows and a columns ($k_1 = k_2 = a$). Levels of factor A should be arranged on whole plots as in a Latin square of $a \times a$ type. Additionally each whole plot should be divided into two subplots ($k_3 = 2$) (as in the Cox type of design). On each whole plot, of the given superblock, one of the levels of factor B with the control B_0 are arranged. This design is repeated t times ($k_0 = bt$). An exemplary superblock has the form which schematically may be expressed as:

A_1 B_0 | B_i	A_2 B_0 | B_i	...	A_a B_0 | B_i
A_a B_0 | B_i	A_1 B_0 | B_i	...	A_{a-1} B_0 | B_i
...	...	...	...
A_2 B_0 | B_i	A_3 B_0 | B_i	...	A_1 B_0 | B_i

The number of treatments (treatment combinations) is equal to $v = a(b+1)$, while the treatment replication vector has the form

$$\mathbf{r} = at\ \mathbf{1}_a \otimes \begin{bmatrix} b \\ \mathbf{1}_b \end{bmatrix} .$$

The association matrices with respect to superblocks, rows, columns and whole plots have the structures:

$$\mathbf{N}_1\mathbf{N}'_1 = a^2t\ \mathbf{J}_a \otimes \begin{bmatrix} b & \mathbf{1}'_b \\ \mathbf{1}_b & \mathbf{I}_b \end{bmatrix} ,$$

$$\mathbf{N}_2\mathbf{N}'_2 = at\ \mathbf{J}_a \otimes \begin{bmatrix} b & \mathbf{1}'_b \\ \mathbf{1}_b & \mathbf{I}_b \end{bmatrix} ,$$

$$\mathbf{N}_3\mathbf{N}'_3 = \mathbf{N}_2\mathbf{N}'_2 ,$$

$$\mathbf{N}_4\mathbf{N}'_4 = at\ \mathbf{I}_a \otimes \begin{bmatrix} b & \mathbf{1}'_b \\ \mathbf{1}_b & \mathbf{I}_b \end{bmatrix} .$$

The strata information matrices, defining properties of the design, are as follows

$$\mathbf{C}_1 = \frac{t}{2}\mathbf{J}_a \otimes \begin{bmatrix} 0 & \mathbf{0}'_b \\ \mathbf{0}_b & \mathbf{I}_b - b^{-1}\mathbf{J}_b \end{bmatrix} ,$$

$$\mathbf{C}_2 = \mathbf{C}_3 = \mathbf{0} ,$$

$$\mathbf{C}_4 = \frac{at}{2}(\mathbf{I}_a - a^{-1}\mathbf{J}_a) \otimes \begin{bmatrix} b & \mathbf{1}'_b \\ \mathbf{1}_b & \mathbf{I}_b \end{bmatrix} ,$$

$$\mathbf{C}_5 = \frac{at}{2}\mathbf{I}_a \otimes \begin{bmatrix} b & -\mathbf{1}'_b \\ -\mathbf{1}_b & \mathbf{I}_b \end{bmatrix} .$$

From the structure of these matrices it results that the eigenvalues of $\mathbf{C}_i$ w.r.t. $\mathbf{r}^\delta$ are:

for C_1: a eigenvalues are equal to 0.5, i.e. $\varepsilon = 0.5$; the remaining ab eigenvalues are equal 0, i.e. $\varepsilon = 0$

for C_2 and C_3 all eigenvalues are equal to 0, i.e. $\varepsilon = 0$,

for C_4: $a-1$ eigenvalues are equal to 1, i.e. $\varepsilon = 1$; $(a-1)(b-1)$ eigenvalues are equal to 0.5, i.e. $\varepsilon = 0.5$; the remaining $a+b$ eigenvalues are equal to 0, i.e. $\varepsilon = 0$,

for C_5: a eigenvalues are equal to 1, i.e. $\varepsilon = 1$; $a(b-1)$ eigenvalues are equal to 0.5, i.e. $\varepsilon = 0.5$; and remaining a are equal to 0, i.e. $\varepsilon = 0$.

It can be checked that this design is also generally balanced. From this the following statements may be inferred:

(i) no contrasts are estimated in the second and third strata;

(ii) all contrasts concerning levels of factor A are estimated in the fourth stratum with the full efficiency;

(iii) contrast between the control level B_0 and the remaining test treatments B_i, is estimated in the fifth stratum with the full efficiency;

(iv) elementary contrasts among test treatments B_i and control are estimable in the fifth stratum with efficiency equal to $(b+1)/2b$,

(v) contrasts among test treatments B_i are estimated in the first and fifth strata with efficiency equal to 0.5;

(vi) a-1 interaction contrasts, including test treatment B_0, are estimated with full efficiency in the fifth stratum; while remaining $(a-1)(b-1)$ interaction contrasts (without B_0) are estimated in the fourth and fifth strata with the same efficiency equal to 0.5.

4. Example

Let us consider a potential field experiment in which we observe the influence of three levels of irrigation (whole plot treatments) and of three levels of nitrogen fertilization (subplot treatments) on the yield of potatoes. Usually, the plots with no irrigation constitute the control as in our potential experiment. From technical point of view it is convenient to use row-column designs with two rows and two columns for the irrigation.

Hence, for the smallest experiment we have a=3, b=3, $k_0 = 3$, $k_1 = 2$, $k_2 = 2$, $k_3 = 3$, $n = 36$. Schematically, a non-randomized plan of design may be expressed as follows:

A_0			A_1		
B_1	B_2	B_3	B_1	B_2	B_3
A_1			A_0		
B_1	B_2	B_3	B_1	B_2	B_3

A_0			A_2		
B_1	B_2	B_3	B_1	B_2	B_3
A_2			A_0		
B_1	B_2	B_3	B_1	B_2	B_3

A_0			A_3		
B_1	B_2	B_3	B_1	B_2	B_3
A_3			A_0		
B_1	B_2	B_3	B_1	B_2	B_3

From this structure it is easy to find the treatment replication vector and the incidence matrices $\mathbf{N}_1$, $\mathbf{N}_2$, $\mathbf{N}_3$ and $\mathbf{N}_4$:

$$\mathbf{r}' = [6\ \ 6\ \ 6\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2],$$

$$\mathbf{N}_1 = \begin{bmatrix} 2&2&2\\ 2&2&2\\ 2&2&2\\ 2&0&0\\ 2&0&0\\ 2&0&0\\ 0&2&0\\ 0&2&0\\ 0&2&0\\ 0&0&2\\ 0&0&2\\ 0&0&2 \end{bmatrix}, \mathbf{N}_2 = \mathbf{N}_3 = \begin{bmatrix} 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&1&1&1&1\\ 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&1&1&0&0\\ 0&0&1&1&0&0\\ 0&0&1&1&0&0\\ 0&0&0&0&1&1\\ 0&0&0&0&1&1\\ 0&0&0&0&1&1 \end{bmatrix}, \mathbf{N}_4 = \begin{bmatrix} 1&0&0&1&1&0&0&1&1&0&0&1\\ 1&0&0&1&1&0&0&1&1&0&0&1\\ 1&0&0&1&1&0&0&1&1&0&0&1\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&1&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0&0&0\\ 0&0&0&0&0&1&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&1&1&0\\ 0&0&0&0&0&0&0&0&0&1&1&0 \end{bmatrix}.$$

The following C-type matrices may be obtained:

$$\mathbf{C}_1 = \frac{1}{9}\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&2&2&2&-1&-1&-1&-1&-1&-1\\ 0&0&0&2&2&2&-1&-1&-1&-1&-1&-1\\ 0&0&0&2&2&2&-1&-1&-1&-1&-1&-1\\ 0&0&0&-1&-1&-1&2&2&2&-1&-1&-1\\ 0&0&0&-1&-1&-1&2&2&2&-1&-1&-1\\ 0&0&0&-1&-1&-1&2&2&2&-1&-1&-1\\ 0&0&0&-1&-1&-1&-1&-1&-1&2&2&2\\ 0&0&0&-1&-1&-1&-1&-1&-1&2&2&2\\ 0&0&0&-1&-1&-1&-1&-1&-1&2&2&2 \end{bmatrix}, \quad \mathbf{C}_2 = \mathbf{C}_3 = \mathbf{0},$$

$$\mathbf{C}_4 = \frac{1}{3}\begin{bmatrix} 3&3&3&-1&-1&-1&-1&-1&-1&-1&-1&-1\\ 3&3&3&-1&-1&-1&-1&-1&-1&-1&-1&-1\\ 3&3&3&-1&-1&-1&-1&-1&-1&-1&-1&-1\\ -1&-1&-1&1&1&1&0&0&0&0&0&0\\ -1&-1&-1&1&1&1&0&0&0&0&0&0\\ -1&-1&-1&1&1&1&0&0&0&0&0&0\\ -1&-1&-1&0&0&0&1&1&1&0&0&0\\ -1&-1&-1&0&0&0&1&1&1&0&0&0\\ -1&-1&-1&0&0&0&1&1&1&0&0&0\\ -1&-1&-1&0&0&0&0&0&0&1&1&1\\ -1&-1&-1&0&0&0&0&0&0&1&1&1\\ -1&-1&-1&0&0&0&0&0&0&1&1&1 \end{bmatrix},$$

$$\mathbf{C}_5 = \frac{2}{3}\begin{bmatrix} 6 & -3 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 6 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & -3 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}.$$

Let us define the set of orthonormal vectors as follows:

$$\mathbf{p}'_{A_0} = 1/2 \quad [1 \ -1 \ -1 \ -1],$$

$$\mathbf{p}'_{A_1} = 1/\sqrt{2} \quad [0 \ 1 \ -1 \ 0],$$

$$\mathbf{p}'_{A_2} = 1/\sqrt{6} \quad [0 \ 1 \ 1 \ -2],$$

$$\mathbf{p}'_{A_3} = 1/2 \quad [1 \ 1 \ 1 \ 1],$$

$$\mathbf{p}'_{B_1} = 1/\sqrt{2} \quad [1 \ -1 \ 0],$$

$$\mathbf{p}'_{B_2} = 1/\sqrt{6} \quad [1 \ 1 \ -2],$$

$$\mathbf{p}'_{B_3} = 1/\sqrt{3} \quad [1 \ 1 \ 1],$$

The efficiency factors of the design with respect to the basic contrast $\mathbf{p}'_j \mathbf{r}^\delta \gamma$ are given in Table 1. From Table 1 we see that nitrogen fertilization effects and interaction effects are compared with full efficiencies (as in complete design). Only, because of incompleteness with respect to whole plot treatments, we suffer the loss of information concerning the irrigation effect comparisons. To improve the statistical properties of irrigation contrast estimators some methods of combining estimators or tests may be applied.

4. Final remarks

The paper deals with two cases of a split-plot type design in which whole plot treatments are arranged in a row-column design. The designs are Cox's type designs with respect to whole plot treatments and complete with respect to subplot treatments (Section 2), complete with respect to whole plot treatments and Cox's type design with respect to subplot treatments (section 3).

In the split plot experiments the comparisons among the subplot treatment effects and among the interaction effects are more interesting than comparisons among the whole plot treatment effects. Hence, the first case of designs con-

Table 1
Strata efficiency factors of basic contrasts defined by vectors **c**.

Vectors **c**	Type	Strata efficiency factors				
		1	2	3	4	5
$\mathbf{c}_1 = \mathbf{r}^\delta \mathbf{p}_{A_3} \otimes \mathbf{p}_{B_1}$	B	0	0	0	0	1
$\mathbf{c}_2 = \mathbf{r}^\delta \mathbf{p}_{A_3} \otimes \mathbf{p}_{B_2}$	B	0	0	0	0	1
$\mathbf{c}_3 = \mathbf{r}^\delta \mathbf{p}_{A_0} \otimes \mathbf{p}_{B_3}$	A_0	0	0	0	1	0
$\mathbf{c}_4 = \mathbf{r}^\delta \mathbf{p}_{A_1} \otimes \mathbf{p}_{B_3}$	A	1/2	0	0	1/2	0
$\mathbf{c}_5 = \mathbf{r}^\delta \mathbf{p}_{A_2} \otimes \mathbf{p}_{B_3}$	A	1/2	0	0	1/2	0
$\mathbf{c}_6 = \mathbf{r}^\delta \mathbf{p}_{A_0} \otimes \mathbf{p}_{B_1}$	$A_0 \times B$	0	0	0	0	1
$\mathbf{c}_7 = \mathbf{r}^\delta \mathbf{p}_{A_0} \otimes \mathbf{p}_{B_2}$	$A_0 \times B$	0	0	0	0	1
$\mathbf{c}_8 = \mathbf{r}^\delta \mathbf{p}_{A_1} \otimes \mathbf{p}_{B_1}$	$A \times B$	0	0	0	0	1
$\mathbf{c}_9 = \mathbf{r}^\delta \mathbf{p}_{A_1} \otimes \mathbf{p}_{B_2}$	$A \times B$	0	0	0	0	1
$\mathbf{c}_{10} = \mathbf{r}^\delta \mathbf{p}_{A_2} \otimes \mathbf{p}_{B_1}$	$A \times B$	0	0	0	0	1
$\mathbf{c}_{11} = \mathbf{r}^\delta \mathbf{p}_{A_2} \otimes \mathbf{p}_{B_2}$	$A \times B$	0	0	0	0	1

sidered (Section 2) is more natural and it may find more applications than the second one, especially in agricultural experiments. The second case of design is considered at least with respect to the symmetry and completeness of theory for the designs considered. It may also find application in experiments in which the size of whole plots is either naturally or technically limited.

The full characterizations of these two kinds of designs with respect to the efficiency factors are given. The properties connected with efficiencies in the strata are very useful in the planning of experiments on a restricted structure of experimental material. From the relationships among stratum error variances, it follows that the treatment contrasts of great interest should be estimated in the stratum as high as possible, preferably in the fifth stratum.

Finally, let us note that the proposed designs are new and the authors do not have real examples. We do hope that desirable practical and statistical properties of designs proposed will cause their common use in practice.

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Układy Coxa w powtórzonym układzie wierszowo-kolumnowym z rozszczepionymi jednostkami i obiektem kontrolnym

Streszczenie

Praca dotyczy powtórzonego układu o jednostkach pojedynczo rozszczepionych, w którym czynnik pierwszego rzędu występuje w układzie wierszowo-kolumnowym. Dodatkowo, wśród obiektów pierwszego rzędu lub obiektów drugiego rzędu występuje jeden obiekt kontrolny.

Przedstawiono dwa warianty układu. Wariant pierwszy dotyczy doświadczenia, w którym jeden z poziomów czynnika A jest poziomem kontrolnym. Zajmujemy się układem, w którym każdy superblok składa się tylko z dwóch wierszy i dwóch kolumn. Dodatkowo zakładamy, że poziom kontrolny spotyka się tylko z jednym z pozostałych poziomów czynnika A, raz w wierszu i raz w kolumnie. Zatem takich superbloków musi być przynajmniej tyle, ile jest poziomów czynnika A. Dla zwiększenia precyzji układ ten powtórzono t razy. Wszystkie poziomy czynnika B rozmieszczone są na poletkach małych wewnątrz każdego poletka dużego. W wariancie drugim poziomem kontrolnym jest jeden z poziomów czynnika B. W takiej sytuacji proponuje się rozmieszczenie na poletkach dużych, poziomów czynnika A zgodnie z zasadą kwadratu łacińskiego. Dalej proponuje się aby każde poletko duże podzielone zostało na dwa poletka małe. Na jednym z poletek małych występuje poziom kontrolny czynnika B, a na drugim jeden z pozostałych poziomów tego czynnika. Liczba superbloków musi być równa lub większa od liczby obiektów rzędu drugiego. Podobnie jak w przypadku pierwszym, dla zwiększenia precyzji układ powtórzono t razy.

Celem tej pracy jest scharakteryzowanie opisanych wyżej układów ze względu na ogólne zrównoważenie. Właściwość ta jest następnie wykorzystana przy badaniu zrównoważenia układu ze względu na efektywność.

Proponowane w pracy układy stosuje się w doświadczeniach, w których jednostki naturalnie można podzielić na dwie części. Przykładem może być podzielenie liścia wzdłuż nerwu głównego na dwie połowy. Układy te stosujemy również w przypadku gdy materiałem doświadczalnym są bliźniacy. Innym przykładem zastosowania mogą być doświadczenia dotyczące organów parzystych (np. nerki), bądź symetrycznych części ciała (np. oczy, ręce, nogi).

Słowa kluczowe: układy wierszowo-kolumnowe, układy niekompletne, obiekty kontrolne, współczynniki efektywności.